1.1 Patterns and Inductive Reasoning

Goals
- Find and describe patterns.
- Use inductive reasoning to make conjectures.

VOCABULARY

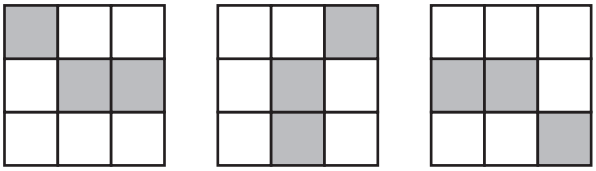
Conjecture

Inductive reasoning

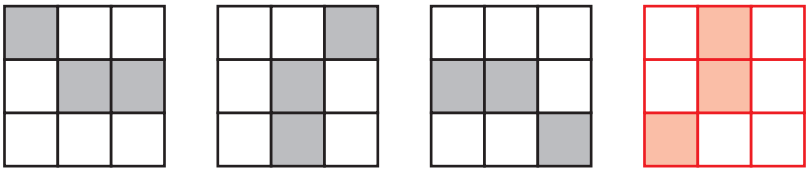
Counterexample

Example 1 *Describing a Visual Pattern*

Sketch the next figure in the pattern.



Solution



How does the shading change from one figure to the next? Is the figure rotated?

✔ ***Checkpoint*** **Sketch the next figure in the pattern.**

1.

Example 2 *Describing a Number Pattern*

Describe a pattern in the sequence of numbers. Predict the next number.

a. 128, 64, 32, 16, … **b.** 5, 4, 2, −1, …

Solution

a. Each number is _____ the previous number. The next number is ___.

b. Subtract ___ to get the second number, then subtract ___ to get the third number, then subtract ___ to get the fourth number. To find the fifth number, subtract ___ from the fourth number.

Answer So, the next number is ____ − ___, or ____.

✔ ***Checkpoint*** **Describe a pattern in the sequence of numbers. Predict the next number.**

2. 4, −20, 100, −500, …	**3.** 10, 20, 40, 70, 110, …

Example 3 *Making a Conjecture*

Complete the conjecture.
Conjecture: The sum of the first n even positive integers is ___?___ .

Solution List some specific examples and look for a pattern.

Examples:

first even integer: $2 = 1(\_\_)$

sum of first **two** even positive integers: $2 + 4 = \_\_ = 2(\_\_)$

sum of first **three** even positive integers: $2 + 4 + 6 = \_\_ = 3(\_\_)$

sum of first **four** even positive integers: $2 + 4 + 6 + 8 = \_\_ = 4(\_\_)$

Conjecture: The sum of the first n even positive integers is _____.

Example 4 ***Finding a Counterexample***

Show the conjecture is false by finding a counterexample.

Conjecture: If the difference of two numbers is odd, then the greater of the two numbers must also be odd.

Solution

Write a difference so that the greater of the two numbers is even.

Counterexample: ___ − ___ = ___

So, the conjecture is _____.

✔ ***Checkpoint*** **Complete the following exercises.**

4. Complete the conjecture based on the pattern you observe.

$$1 = 1$$

$$1 + 2 = 3 = \frac{2(2 + 1)}{2}$$

$$1 + 2 + 3 = 6 = \frac{3(3 + 1)}{2}$$

$$1 + 2 + 3 + 4 = 10 = \frac{4(4 + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 = 15 = \frac{5(5 + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 + 6 = 21 = \frac{6(6 + 1)}{2}$$

Conjecture: The sum of the first n positive integers

is _____.

5. Show the conjecture is false by finding a counterexample.

Conjecture: The difference of two negative numbers is always negative.